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TECHNICAL NOTE

Rodolfo G. Pregliasco,[1] Ph.D. and Ernesto N. Martínez,[1] Ph.D.

Gunshot Location Through Recorded Sound: A Preliminary Report*

ABSTRACT: Using a video tape and three still photographs, we analyze recorded sound to attribute authorship in a fatal shooting incident.

We measure the acoustic signature of the scene with four test detonations, and we discuss two methods of processing putative echoes in the audio band of the video. They allow us to locate 11 out of 17 shots, with a precision of $0.4 \times 1.0 \text{ m}^2$.

Combining the location of the shots with the known positions of the participants, we arrive at the clear attribution of eight shots; three other shots (among which is the fatal one) probably issue from the same shooter, who remains unidentified within a small group of men. Our results lead to the exculpation of the main suspect.

We analyze the statistical significance of the results, use Monte Carlo simulations to set an upper bound to the probability of false positives, and discuss areas of improvement of the method.

KEYWORDS: forensic science, forensic physics, video recording, gunshot wounds, acoustics, echolocation, computer simulation, Monte Carlo method

Gunshots can be located by their sound (1). In a famous instance, the appearance of an audio tape inadvertently recorded around the time of President John F. Kennedy's assassination in Dallas led to complex studies on the acoustics of the case (2). This approach remained fruitless (3,4), but has been brought to life again by D. B. Thomas's recent reevaluation of the evidence (5).

As a rule, acoustic studies need recorded sound. One exception to this rule is Luis W. Alvarez's acoustic analysis of Abraham Zapruder's *silent* 8-mm film of the Kennedy assassination (6). What is noteworthy is that Alvarez based his acoustic work on *graphic* clues: He measured the angular acceleration in Zapruder's tracking of the presidential motorcade and related its sudden peaks (as shown by the extension of highlights into *jiggles*) to the incidence on the camera of shock waves from the supersonic shots. Anyway, if filming an incident was a rare coincidence in 1963, it is becoming commonplace due to the proliferation of video camcorders, which record sound as well. In this paper we report on our successful acoustic location of the origin of gunshots in a complex real case.

During the repression of labor riots in Cultural Có, a town in Argentinian Patagonia, in April 1997, a 24-year-old woman was mortally shot. According to all available evidence, she was a passerby, not involved in the confrontation.

The bullet that killed her was a flattened lead slug without any rifling marks. The details of this slug's deformation, its low energy on impact (shown by the fact that after severing the right carotid artery, it was stopped by, and caused minimal damage to, the fourth cervical vertebra), and some silica inclusions it contained led to the conclusion that it was a bullet that had ricocheted tangentially from the ground. Probably as a consequence of the rebound it lost its brass jacket, together with any rifling marks that might have identified the weapon that shot it. From its weight and conserved dimensions, the projectile was compatible with the lead-antimony kernel of a 9×19 mm projectile, the standard issue for the police forces on the scene.

The incident had been taped on video, and acquired instant celebrity when it was shown in the national news. The relevant section of the tape lasts 16 seconds and shows over two dozen policemen charging across a bridge from north to south into a hail of stones, the front rank protected by plastic shields. Some policemen brandish guns or batons, and 17 shots are heard over eight seconds. The gunshots were identified as such and their onsets placed on the audio recording by the Scopometric Division of the Argentine Federal Police, the local equivalent of the FBI.

Midway through the shooting, a silhouette in the background is seen to stagger; clearer photographs show her to be the victim (see Fig. 1). The photographs were taken by reporters who stood close to the cameraman, near the North end of the bridge. The incident is narrated into a hand-held microphone by a journalist who stood at the left of the cameraman.

Repeated studies of the tape and photographs gave no firm clues on who fired the shots, in particular the one that killed the woman. On the basis of one of these studies, one policeman was charged with manslaughter. Two years after the crime, the investigating judge asked us to perform a new study of the material from our point of view as physicists. Like the United States, Argentina has a

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[1] Centro Atómico Bariloche, CNEA and Instituto Balseiro, UN Cuyo, 8400 Bariloche, Argentina.

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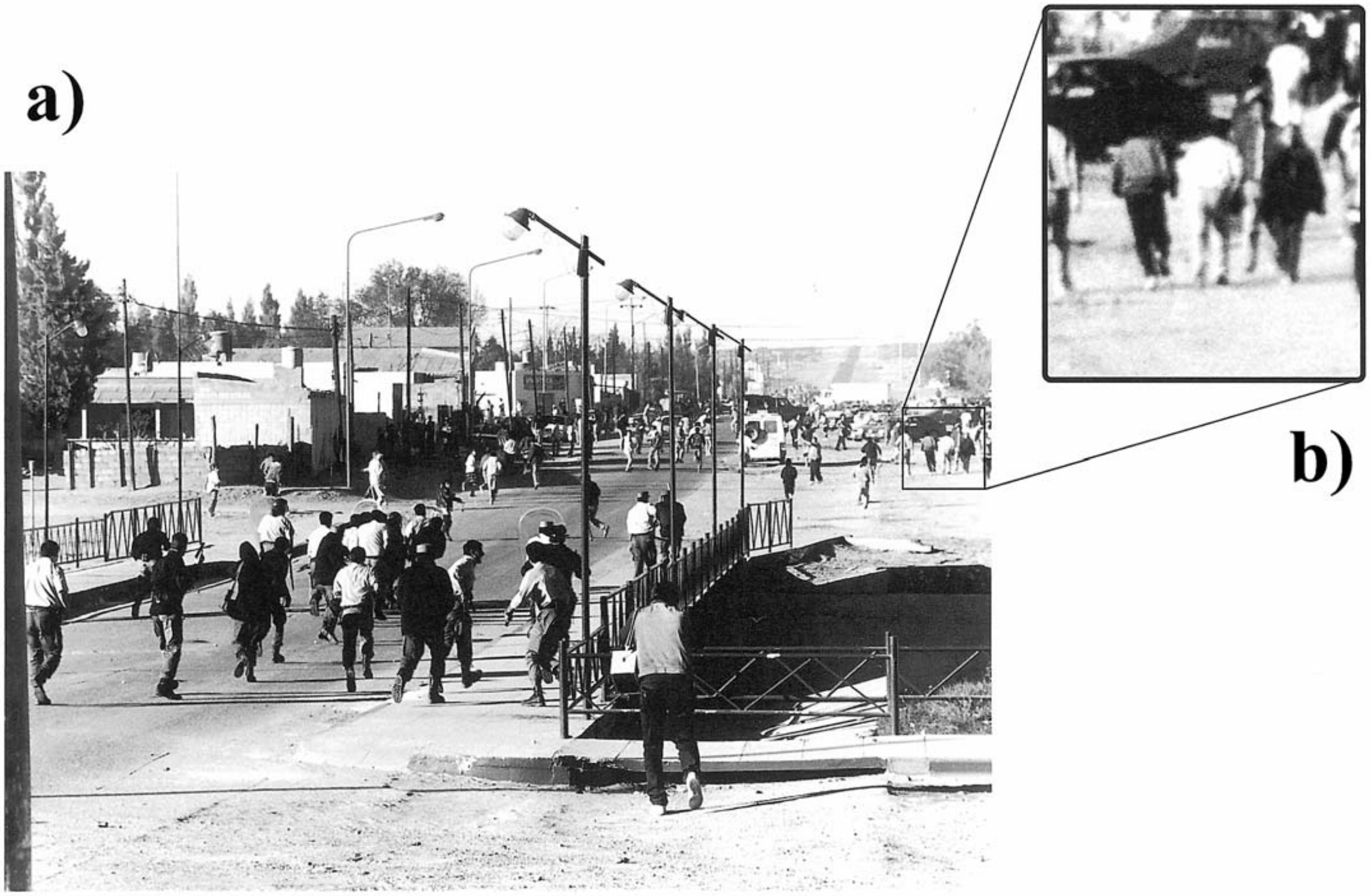

FIG. 1—*a) Second of three still photographs (originally in color) shows the charge of police forces across the bridge. Half of them are in their shirt sleeves, most wear no headgear. b) This detail shows the victim, at right in dark clothes, walking towards the camera. This picture was taken approximately half a second before she was hit.*

federal law system. Under the legal system in the province in question, one judge investigates the case and formulates the accusation, while three different magistrates judge the matter.

The grainy tape gave us too few visual clues to determine the source of the gunshots. We turned then to the audio band of the tape. The incident had been recorded according to the PAL norm in a standard VHS video cassette with a Panasonic 9500 analog video camcorder. The sound was recorded by a hand-held general purpose monoaural directional microphone, a Shure BG 1.0. As the microphone's frequency response extends only to 12 kHz (according to the manufacturer's specifications), it is the weakest link in the quality of the recorded sound. The analog recording of the sound was digitalized at a rate of 44 100 points per second, with a resolution of 16 bits. We worked on the digitalized version.

In Fig. 2 we show an *audiogram* of one of the shots. The shape of this audiogram is quite different from the high frequency recordings of gunshot detonations in laboratory conditions (7). On one hand, although a gunshot detonation is rich in very high frequencies, the limited dynamic response of the commercial recording equipment filters the high frequencies out of our audiograms. Needless to say, this limited dynamic response is much more typical of real recordings than the high-frequency experimental one. For a study of gunshot recordings with nonspecialized equipment, see Koenig et al. (8). On the other hand, the pulse from the gunshot itself makes up only the beginning of the audiogram, the decreasing tail is made up of *reverberations*, sound from the shot that reaches the microphone after one or more scatterings. Considering the distances from the shots to the microphone that we will determine later, the first echo, from the paved ground, reaches the microphone roughly 0.2 ms after the direct sound. Within the tail of reverberations, some clear secondary peaks are seen after each shot.

Methods

Acoustics at the Crime Scene

We suspected that at least some of the secondary peaks were echoes from stationary objects at the crime scene. To investigate the acoustics at the scene we recorded detonations at four selected locations on the bridge.

The recorded waveform of a gunshot is a convolution of three distinct factors: The gunshot pulse (high frequencies), the response of the recording equipment, and finally the acoustic response of the scene. Experimental studies (8) have shown that the use of nonspecialized recording equipment can alter critically the shape of the initial pulse. Given enough distance between the muzzle of the gun and the objects that surround it, the acoustic response of the scene will influence only the long time, low frequency, features of the waveform. This separation of short-time and long-time features allowed Koenig et al. (8) to focus their study on the convolution of gunshot pulses with the recording equipment by restricting their analysis (and choosing their locale) so that echoes could be reasonably excluded.

It should be kept in mind that in this study we are not trying to identify gunshots as such, or even less to discriminate between weapons or find their orientations during shooting. Rather, we are trying to investigate the echoes produced by the surroundings, that is the long-time features of the recorded signal. Given our modest aims, any sharp delta-like click or detonation can replace the original gunshots. This is the physical reason why we decided to use firecrackers for our experiments on the scene. Nonphysical reasons were expediency, lower costs, and social tact: The neighborhood was very sensitive to any display of police force.

Accordingly, we detonated powerful firecrackers at a height of 2 m, suspended over four well measured positions on the ground.

The original equipment was no longer available for our experiments on the scene. As the microphone had the most limited frequency response in the recording equipment, we took care to use the same model, a Shure BG 1.0. We recorded the sound with a sound card integrated in the motherboard of a Texas Instruments Extensa 610CD laptop computer. The digital sound processor characteristics are: Resolution 16-bit, sampling rate up to 44.1 kHz, aliasing filter in the range of 20 Hz to 22 kHz (±1 dB). A laptop computer offers an advantage over tape recorders or video cameras, as it allows on-the-spot checking of the audiograms.

We placed the microphone near the camcorder's original position. In a clue-rich urban landscape the viewpoint of any image can be located with a precision of roughly 10 cm, by careful alignment of sharp features, even without the help of a surveyor's theodolite.

We also measured all relevant distances and corrected the plan we had been given, as regular features like lamp posts are never exactly equidistant.

The waveforms we recorded on the scene were similar in appearance to the one shown in Fig. 2, and they also showed a set of well-defined secondary peaks, which changed their positions as the place of detonation varied.

If one knows the microphone's location and the origin of the sound, it is elementary to calculate where the echo produced by a particular object should be in the audiogram. The inverse calculation, that is, given an echo finding which object produced it, is not possible. That is because the echo gives information only on its *delay* with respect to the direct sound. One single number does not locate a point on a plane; mathematically, a constant delay defines not a point but an ellipse whose focuses are at the microphone and the detonation (Fig. 3).

To find the locations of the objects that produced echoes we superposed the ellipses corresponding to the most nitid echoes for four detonations (with two detonations there are multiple solutions, as two ellipses cross on two or four points; we used four detonations to have some redundancy). In Fig. 4 we show the set of ellipses for the four detonations drawn on a scale plan of the crime scene, distinct echoes are produced by eight steel lamp posts on the sides of the bridge (six of them are visible in Fig. 1), several utility posts and road signs that did not exist on the day of the incident, and two walls in the left background (not shown).

The special situation of the crime scene, an isolated bridge, results in a poor *acoustic signature*, with few distinctive features. Instead of buildings or walls that could produce strong peculiar echoes, we had an array of thin posts that gave weak (but sharp) echoes, all alike.

Once we knew which objects were producing echoes, we noticed that there were fluctuations in their times of arrival. We detected a systematic drift, which turned out to be compatible with the warming of the air as our measurements progressed along the morning, from 17.2°C to 19.0°C. We measured the temperature as we worked, in the shade, at heights of 3 cm and 2 m. Once we discounted this drift, random fluctuations remained. They have an approximately gaussian distribution, with a null mean value and a root-mean-square value of 1.1 ms. The main source of these fluctuations is probably the lack of definition in the onset of the echoes in the audiogram, with perhaps some contribution from thermal turbulence in the air.

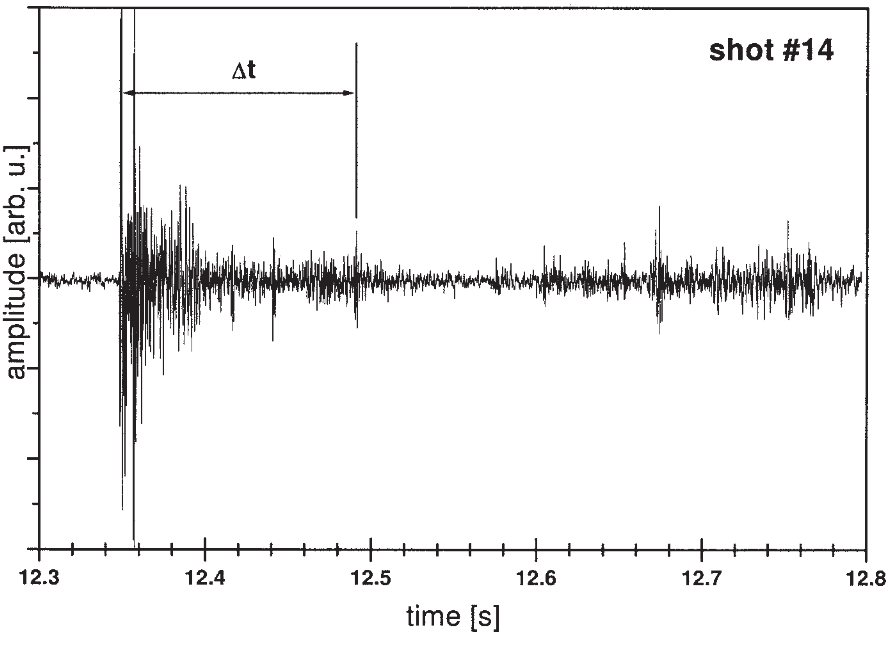

FIG. 2—*Audiogram, a graph of sound intensity (in arbitrary units) as a function of time (counted in seconds from an arbitrary origin).* Δt *is the time delay of one echo from the recorded start of the shot, in this case the 14th.*

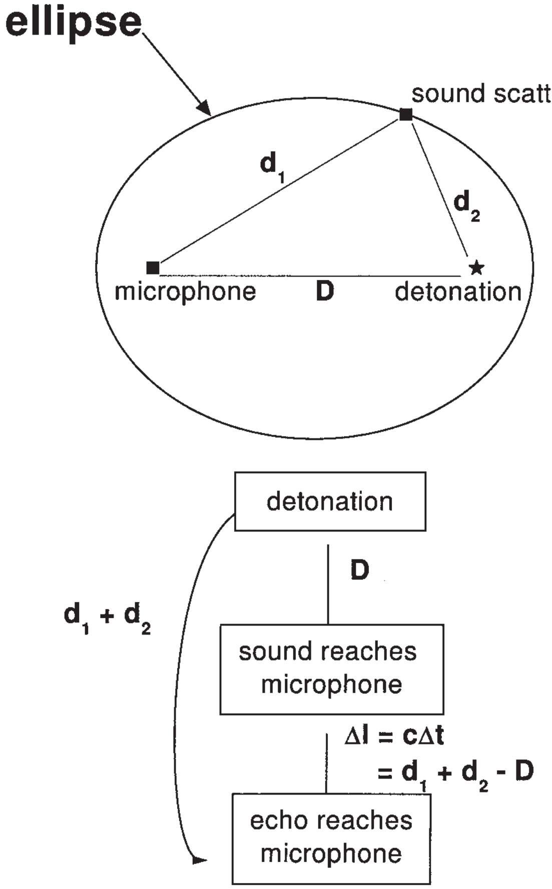

FIG. 3—*Meaning of the measurements on the bridge. If the positions of the detonation and the microphone are known, the fact that an echo is recorded with a given delay* Δt *is not enough to locate the sound scatterer. If the speed of sound is* **c,** *the delay means that the indirect distance detonation-scatterer-microphone was* cΔt *longer than the direct one, detonation-microphone, thus the possible scatterers lie on an ellipse with its two focuses on the microphone and the detonation.*

Locating and Timing the Shots

We have seen how detonating firecrackers on the bridge showed us which objects produced the strongest echoes. The next step was to locate the actual shots. In principle, this is the inverse of the problem we solved in the last section. That is, knowing where the microphone was located, what object produced the echo (from our experiments on the bridge), and the echo delay (from the audiogram), now we wanted to find the origin of the shot.

Actually, from the images in the video we know accurately the camcorder's position; the position of the microphone must be inferred. This is not a critical problem, from the known facts that the journalists were huddled against a wall on their right, and that the speaker stood next to her cameraman, we estimate the hand-held microphone's position at 1 m to the left of the camera. Due to the geometry of the situation, the final locations of gunshots are quite insensitive to the exact placement of the microphone.

Again, one echo is not enough to locate a point. It only defines a curve of constant delay, an hyperbola in this case (Fig. 5). While all points on this hyperbola are possible origins of the shot, only one is the solution. But if we draw the hyperbolas corresponding to each of the echoes produced by each of the scattering objects, they should all intersect on the solution.

However, several practical factors complicate this textbook solution. First, as all posts produce similar echoes, we cannot assign an echo to a post: Echoes carry no tags. Second, not all echoes were present (or detected, rather), as people in the scene could absorb sound, before or after scattering at the posts, or otherwise clear echoes could be obliterated by louder noises. Third, there are peaks in the audiogram which resemble echoes. They can be either real echoes from people or other moving objects (notably plastic shields in a favorable orientation), or independent sharp noises. It should be kept in mind that as gunshots and their echoes are very short, Fourier analysis is of little use in characterizing them; we are currently experimenting with *wavelets* in echo recognition (9).

Faced with this difficulty, we decided to gather a list of possible echoes for each shot. Heeding the criticism by the Committee on Ballistic Acoustics in the case of J. F. Kennedy's assassination (4), we chose the echoes by *a priori* criteria. The peaks we considered were very sharp spikes in the same direction as the shot, with a similar rise time of less than 0.1 ms. They had to be at least twice the height of the surrounding noise to qualify as a possible echo. This produced lists of 15 to 20 possible echoes for each shot. As some of the shots happen in rapid succession, the same echo could appear in more than one list.

An important exception to our remark that echoes carry no tags was provided by the two walls in the left background. As these two distant walls are practically in line with the camera and the general

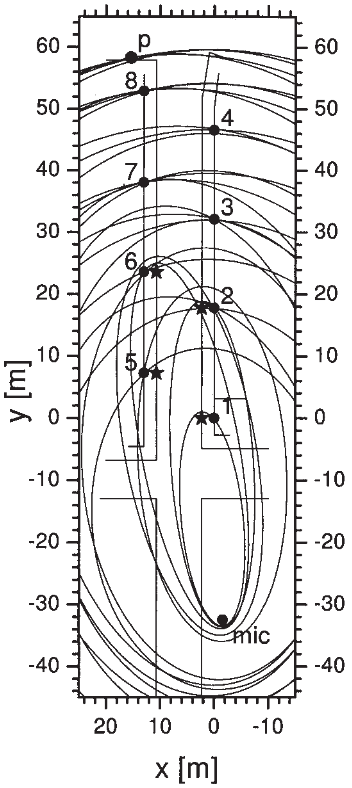

FIG. 4—*Results of the measurements on the bridge. Scale plan of the bridge, the numerals from 1 to 8 denote the steel lamp posts, six of which can be seen in Fig. 1. The stars show the four test detonations, while "mic" places the microphone. The ellipses for the echoes of the four test detonations, drawn according to Fig. 3, cross on the positions of the scattering objects, which are found to be the lamp posts, some other objects that were not present on the day of the shooting (like the utility post just above lamp post 8), and two walls in the left background. These last ones have not been included in this drawing.*

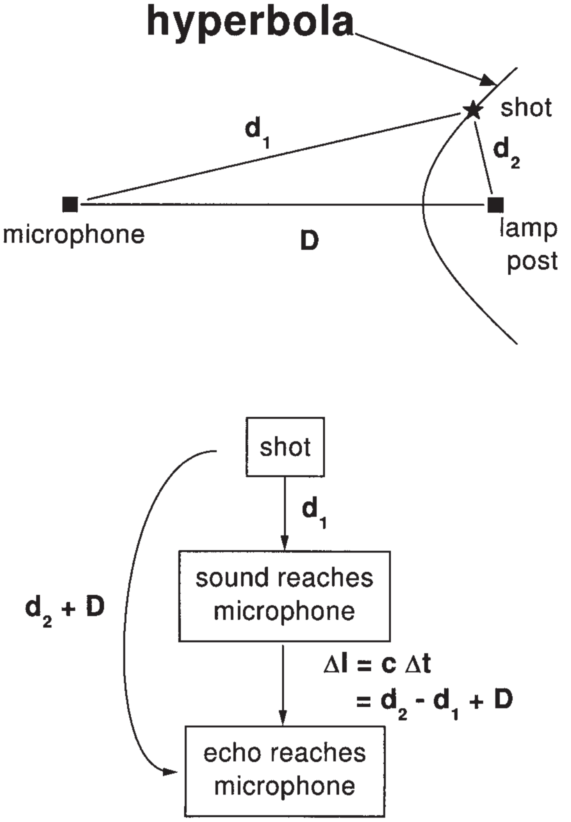

FIG. 5—*If the positions of the scattering object (the lamp post in this example) and the microphone are known, the fact that an echo is recorded with a given delay* Δt *is not enough to locate the shot. With a speed of sound* **c**, *the delay means that the difference of the distances shot-lamp post minus shot-microphone was a constant* **(c**Δt-**D)**, *thus the possible shots lie on a hyperbola with its focuses on the microphone and the scattering object.*

scene of the shooting, and they are situated over 120 m away, their two echoes reach the microphone with a constant separation, 119 ± 1 ms. Thus, they form an easily recognizable pattern, sometimes visible with the naked eye, and in more difficult cases detectable with a coincidence method. Even so, we could not detect this pattern for all shots, as in some cases it might have been covered by a following shot.

To search for the location of each shot we used two methods that are mathematically equivalent, but differ vastly in their visual impact.

The first method consisted of computing the matching between one given list of delays of echoes and those that would be generated by the posts and walls we had identified by a shot produced on a given position (x, y). We did this by computer simulation. We created a grid over the area of interest (which covered the bridge where the police forces ran and the streets beyond, where the rioters fled, we took care to make no presuppositions about the origin of shots) and generated a shot at each point of the grid. We computed the echo delays and compared them with the list. We deemed that two echoes coincided if they came within a certain window, usually 1 ms in width, or narrower. We added the coincidences into a *coincidence function,* which we graphed on the plan of the area. We enhanced the coincidence function by giving extra weight to the double echo from the two walls, when we could find it. A clear isolated maximum of the coincidence function located the most probably origin of the shot. We adjusted the grid spacing along the calculation to achieve good definition and reasonable speed of computation.

It is important to stress two points about this coincidence function. First, it is purely heuristic; after some experimenting with parameters it gave good results with our control cases, and located well the test detonations (see further on). But as similar functions also gave satisfactory results, the function's exact expression is not important, so we will not go into details. Second, this coincidence function should not be confused with a correlation function of two wave forms; our lists of delays contain no information about the waveforms of shots or echoes, other than their onsets.

The second method was based on the crossing of hyperbolas we discussed above. As we could not know which echo to assign to each post, we drew hyperbolas for the whole list as if they had all been reflected by each one of the scattering objects, even though only one, or none, could have been. The extra hyperbolas crossed each other in a random fashion all over the area; only the correct ones met in one point. We took the precaution of drawing with different colors all the hyperbolas assigned to the same post: As each post can produce only one echo for each shot, at the correct crossing the hyperbolas that cross must have different colors. As we said above, echoes carry no tags; in this approach, each echo acquires a meaning as such and it is assigned to a given object, only through its meaningful position in the whole pattern. The rest are discarded as spurious. This method allowed us to judge at a glance the general situation, and accordingly it was more useful in finding out the overall pattern of echoes. The result for one of the actual gunshots is shown in Fig. 6; a detail of the surface of the bridge, with the crossing of the correct hyperbolas. The maximum of the coincidence function (not shown here for the sake of a clearer picture) coincided with this crossing.

To test the methods, we applied them to our experimental detonations. Both methods gave clear concurrent results, placing all explosions within regions of roughly 30 cm along the bridge, and 60 cm across. We discuss the application of these methods to the real case in the section on *Results.*

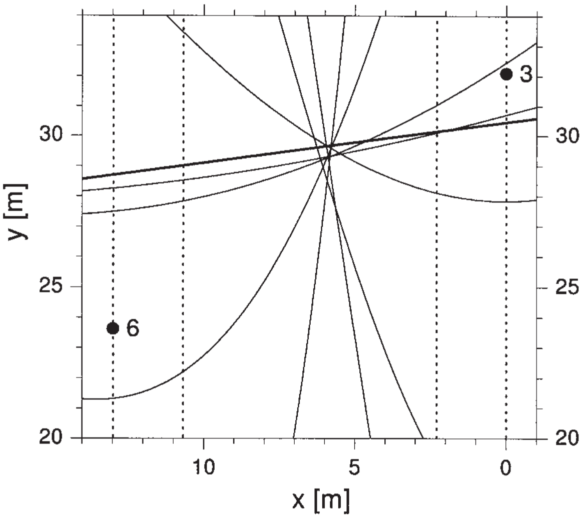

FIG. 6—*Crossing of hyperbolas for an actual shot. Only the correct hyperbolas are shown, that is, those given by the correct combination of a post and its own echo, plus the double hyperbola from the two walls in the background. They locate this shot at* $x = 5.8 \pm 0.2$ *m,* $y = 29.5 \pm 0.5$ *m. The coincidence function is not shown here for the sake of a clearer picture, but its maximum coincides with the crossing of the hyperbolas.*

FIG. 7—*Comparison of a detail in photograph #3 with details from four consecutive fields from the digitized video. As the video has been digitized at 25 frames per second, the fields, composed of odd and even horizontal lines respectively, here denoted by "a" and "b," are separated by 20 ms. As the boy at the left of the man in light clothes runs into the shade, his right arm is progressively obscured. The photograph corresponds closest to field 338b, hence was taken at 16.04 s; the adjoining fields are definitely too early or too late.*

Only after locating a shot could we time it accurately, because we had to discount the time it took for the sound to reach the microphone. Previous studies had not taken into account this delay, and as a result had looked for graphic signs of shooting about five frames too late in the video.

Attributing the Shots

After locating and timing the shots, it was necessary to find who was in that location at that time. Placing objects from a picture is a problem solved by photogrammetric techniques.

Although we could locate most protagonists on the scene analyzing the three still pictures, not every man could be placed in this way. When they run in a group, only some men in the outside could be definitely placed; the positions of the ones inside remain undetermined, but of course bounded by the ones on the outside.

Timing the Still Pictures

As we had a changing, dynamic scene, we needed to place the three still pictures within the video recording. We digitalized the video at a rate of 25 frames per second. Each frame can be deinterlaced into two fields, which gave us video pictures 20 ms apart: If we could pair a still picture to a video field unequivocally, it would time the picture to within 10 ms. Doing this proved to be surprisingly easy. We paid attention to the swinging of arms and legs when they go through the vertical, as their maximum angular velocity makes them specially sensitive to time changes. In at least one case, a person runs into the shadow of a wall; as shown in Fig. 7, the occultation of his right arm allowed us to time the picture with precision.

It turned out that the still pictures frame the shooting very neatly. The first one had been taken just before the first shot, the second one around the time of the ninth one, and the third some seconds after the last shot, but when the policemen were still running.

Steady Running

Once we had determined the positions of most policemen in the three still pictures, and we had timed them, we plotted the *y*-coordinates of the policemen, that is their positions along the bridge, as a function of time. We could locate 14 of the men in all three pictures. They ran at a steady rate: The three points for each man fell on straight lines, with Pearson coefficients of correlation above 0.99 in ten of the cases, and in no case below 0.96. The conclusion is that almost all these men were running at constant speeds during the 10-second period covered by the photographs. Not all of them ran at the same speed, however; while the men in the front rows jogged at about 2.5 m/s, the men at the back tried to catch up at 3–4 m/s. As the men were obviously running at constant speed, we interpolated linearly to find their positions at intermediate times.

There are a few men who pirouette, dodge or dash suddenly; we corrected their positions one by one, using as references their steadier companions. As we said above, for several of the men we could obtain no quantitative position estimations, so we limited ourselves to bracketing them by their measured colleagues.

As a result of this procedure, we put together a plan of the individual positions of the policemen at the times when each shot was generated. We identified most policemen by arbitrary numbers. Those we could not locate individually were not numbered, specially three or four in a group at the right of the front rank.

We estimate that the uncertainties in the transversal direction (across the bridge) are negligible. They are more serious in the longitudinal sense (away from the camera); for policemen near the camera the expected error amounts to 1 m or less, while for the men far away the error could reach 2 m.

Results

When we applied the methods explained above to the real shots, we could locate 11 out of the 17. For all of the 11, at least 7 of the 10 echoes (8 posts and 2 walls) were accounted for. The remaining six shots gave wide undefined maxima in the coincidence function, and not enough hyperbolas crossed neatly enough for a positive location.

After we completed the two distinct phases, location (and hence consequent timing) of shots, and location of protagonists at those same times, we proceeded to combine them. We stress this order; in no case did we try to find the dubious location of a shot by looking at the possible shooters.

In Fig. 8 we illustrate this procedure with shot #14, which we used as an illustration in Fig. 6. After finding the probable origin of the shot with both methods, we discard the spurious hyperbolas and show only the correct ones and information from the coincidence function. The probable origin of the shot is localized within an area about 40 cm deep (in the direction along the bridge) and 100 cm wide (across the bridge). This precision was quite sufficient to enable us to correlate the location of the source of the gunshot with the location of individuals identified in the videotape. In the figure we have superposed the extrapolated positions of the men at the time the shot was produced. The policeman numbered as "18" is the likeliest candidate. In Fig. 9 we show a detail of the video frame nearest the shot; as policeman 18 is holding his pistol in a suitable position for shooting into the air, there is no reason to doubt that he fired shot #14.

Shot #14 has no special significance in the actual case. We have used it as an illustration of the method merely for its didactic value: All the steps in its study are clear, specially the detail of the video frame shown in Fig. 9, which can stand the reduction in size and loss of color of the printed page.

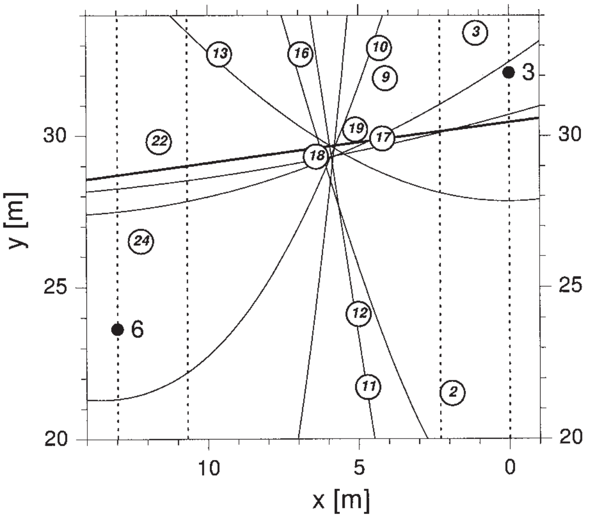

FIG. 8—*Attributing responsibility. We superpose the location of the policemen at the time of the shot to Fig. 6, which corresponds to shot #14. The policeman identified with the number 18 is the obvious candidate.*

FIG. 9—*Confirming opportunity. A detail of the frame of the video tape (originally in color) that corresponds to the firing of shot #14 shows policeman 18 with raised pistol.*

TABLE 1—*Location of shots and their authors.*

	Time [s]		Position (x [m]; y [m])‡		
Shot #	Recorded*	Actual†	Shot	Nearest Policeman	Nearest Policeman #
1	5.814	…	…	…	…
2	6.161	…	…	…	…
3	6.373	6.264	(1.6; −0.2)	(1.7; −0.6)	5
4	6.473	…	…	…	…
5	7.761	7.587	(8.8; 20.7)	(8.8; 20.2)	13
6	8.195	8.082	(1.9; 0.5)	(1.8; 0.2)	23
7	9.176	9.049	(1.0; 5.7)	(0.9; 6.0)	6
8	9.955	9.768	(5.0; 26.3)	…	In group
9	10.299	10.109	(6.4; 26.0)	(6.0; 26.0)	18
10	10.479	…	…	…	…
11	10.512	…	…	…	…
12	10.806	10.613	(5.1; 28.3)	…	In group
13	10.996	10.801	(4.7; 28.7)	…	In group
14	12.348	12.153	(5.8; 29.5)	(6.4; 29.3)	18
15	12.797	12.595	(6.2; 31.4)	(6.5; 31.4)	18
16	13.081	12.918	(5.8; 17.9)	(4.6; 17.8)	5
17	13.999	…	…	…	…

* The onset of recorded shots can be timed with a precision of 0.0001 s, but in this table we express results with a precision of only 0.001 s (see main text for details).

† The actual time when a shot was fired is obtained subtracting the travel time from the recorded time. The uncertainties in the location of the shot result in an uncertainty of roughly 1 ms in the time of firing.

‡ The coordinates are across the bridge (x) and along the bridge (y). The uncertainties for the locations of shots are roughly ± 0.5 m for x, and ± 0.2 m for y; for the locations of policemen they are basically a man's arm reach, some 0.8 m in the x-direction. In the y-direction they are given by the errors in photogrammetry discussed in the text, and vary from 1 m for small values of y, to 2 m for the high values.

Table 1 shows the results for the 17 shots. To measure positions, we place the *y*-axis along the right (West) side of the bridge, with its origin at the start of the bridge. The *x*-axis extends at right angles, to the East. The bridge is 13 m wide. The times are measured from a conventional beginning in the tape; there is nothing special at this moment, we choose it merely to maintain the notation of previous studies.

A word on experimental errors. While the onset of a recorded shot can be placed with a precision of 0.1 ms, the time of origin has an error nearer 1 ms, mainly due to the uncertainty in the coordinate *y* (an error of 34 cm in distance translates to an error of 1 ms in time). Accordingly, we have rounded all times to the nearest millisecond. The estimated errors for coordinates are shown on the table (10,11).

For eight of the localized shots, their origins coincided with a man who was obviously holding a gun which could be fired. In one of the cases the origin of the shot was compatible with the positions of two of the policemen; as one of them was holding his empty hands in plain sight, he could be excluded. The other three shots, numbers 8, 12 and 13, were well located but issued from the middle of a group at the right of the front rank, an indistinct cluster of dark shapes.

Discussion

One result of our analysis is clear: Although at the beginning of the case the lawyers for the policemen claimed that their clients had merely waved their weapons but had never fired them, all the shots we could locate came from the police ranks. In a revised version of their position, the policemen admitted firing, but claimed it was only in response to initial shots by the rioters. As the first two shots remain unlocated, any one of them could have come from the rioters, so in principle the defense claim is possible.

Careful analysis of the video tape proves that the victim, who was walking briskly towards the camera, started falling roughly at the time of the ninth shot. A calculation of the time of flight of the projectile shows that the ninth shot, and all following ones, could not have reached her before she started falling, and so can be discarded.

As we said above, the victim was hit by a shot that bounced off the ground. The authors of the seventh, sixth and fifth shots are individualized, their weapons clearly seen, and they are all shooting into the air. As the fourth shot could not be located, nothing is known about its direction. However, it happens 3.4 s before the victim starts falling. After the shot severed her right carotid artery, the sudden fall of blood pressure in the brain would make it highly unlikely that she could have maintained her quick energetic walk for over three seconds. This argument applies even more to the unlocated first and second shots. As a result, our conclusion (shared by all the other experts) was that the victim probably was hit by the eighth shot.

This leads us to the second result of our analysis, which is probably the most important: The suspect who had been charged on the basis of a previous study fired none of the 11 located shots, and in particular he did not fire the 8th shot.

As explained above, the 8th shot, along with the 12th and the 13th, came from a cluster of men at the right of the front of the charge. It should be noted that the origins of these three shots move in time at the exact speed of the men who frame this group. This is clearly shown in Fig. 10, where the *y* coordinates of the three shots (their *x* coordinates coincide within the experimental uncertainty) are graphed as functions of time, and compared to the interpolated positions of two policemen who framed the group. This leads us to believe that these three shots were fired by the same person.

In sum, although we still cannot say who fired the fatal shot, our acoustic analysis has exculpated one suspect, and circumscribed the responsibility to one man within a small, well localized group.

It should be noted that we have not used the loudness of the shots in our analysis, although it is one of the most tempting clues for localization; everybody knows that the intensity of sound decreases as the inverse square of distance. However, after some study we came to the conclusion that intensity was an unreliable magnitude for analysis. First, as the directional microphone was being held in the hand of the reporter who narrated the incident, considerable

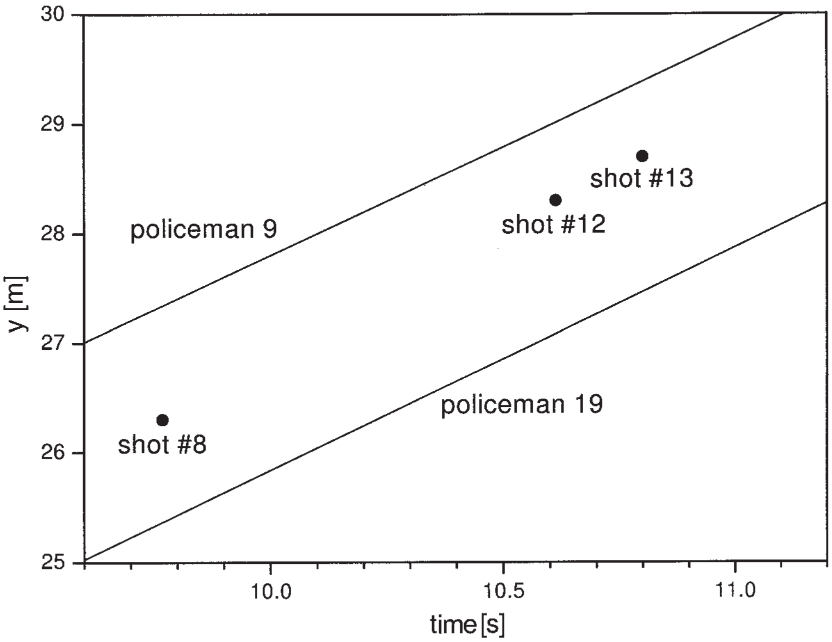

FIG. 10—*Graph of positions along the bridge (y) as functions of time. The three "shots from the group" are framed by the (interpolated) positions of policemen 9 and 19. The shots seem to have been fired by one unidentified author who moved at the same speed as his companions.*

variations in intensity from shot to shot could be due to simple motion of the microphone. Second, the microphone itself has a limited dynamic response, it could be saturated by high intensity detonations, or the high intensities could escape the dynamic range of the camera. All these were variables we had no access to. In addition, it should be kept in mind that the inverse square law holds only for laboratory conditions. Our experiments on the bridge showed a very confusing relation between distance and intensity.

Statistical Significance of Results

The physical aspects of our treatment are well known and require no validation. The use of a pattern of echoes, however, though straightforward in principle, should be checked in practice.

We tried to apply the most strict scientific methodology; for each shot first we gathered a list of possible echoes, then we located an origin for the shot, and last we added the positions of the policemen. The fact that all the shots we could localize coincide with policemen who were raising pistols at the time speaks for the reliability of the method. The only exceptions are the trio of shots #8, #12 and #13, which acquire special relevance if #8 is indeed the fatal one.

We explored the probability of false positives with Monte Carlo simulations (12,13) applied to location via the coincidence function. We found that as long as at least six echoes coincided, the probability of an error of more than 60 cm was well below one in a thousand.

Actually, this is an upper bound; three procedural reasons make the probability of a false positive much smaller than this. First, we located the shots using both the coincidence function and the intersection of hyperbolas; as this last method requires human judgement, it cannot be automated satisfactorily. Second, in actual practice we used the echoes from the background walls as a pattern, either both were recognized or none, whereas in our Monte Carlo simulations they acted independently. Third, all our shots were located by at least seven echoes. In the simulations, when there were seven coincidences there appeared no instances of errors greater than the separation between grid points.

In view of the very small probability of a false positive, we are confident that each one of the shots 8, 12 and 13, even considered independently, has been reliably located. Taken as a pattern, the trio of shots which appear at places that move at the same speed as the surrounding men are beyond any reasonable doubt. Our conclusion is that very probably the victim was killed by the #8 shot, and that this shot was fired by a man within the cluster at the right of the front rank.

Possibilities for Improvement

The areas in which the methods can be improved are clear. First of all, echo identification: We believe that wavelet analysis is the most promising approach, and we are working on it.

Second, after promising echoes have been isolated, they have to be combined into a coherent pattern. This is a mathematical problem: We are trying to automate into algorithms the procedures we used to find the sources of sounds, and at the same time maintain a firm human control of decisions.

We are confident that these goals are attainable, and that refined versions of our method will prove of forensic help in future cases. Even with our ad hoc methods, this case is an example of the information that can be obtained from the sound in a film. One of the main advantages of sound is that information comes from a very wide angle and is not limited to the line-of-sight scene. One can only speculate on how much work and argument, not to speak of suspicion and ill-feeling, would have been saved if Abraham Zapruder had been able to record sound in Dealey Plaza. With video cameras in widespread use, this kind of evidence will become more common every day.

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Additional information and reprint requests:
Ernesto N. Martínez, Ph.D.
Centro Atómico Bariloche
8400 Bariloche
Argentina
E-mail: emartine@cab.cnea.gov.ar